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OPTIMIZATION OF THE PRODUCTION PROCESS OF A BAKERY ENTERPRISE BASED ON THE CP-SAT METHOD

T.Zh. Mazakov[1,2], Sh.A Jomartova[1]✉, A.T. Mazakova[1], T.T. Duzbaev[1], A.D. Mailybaeva[3], B.Zhao[4]

[1] *Al-Farabi Kazakh National University, Almaty, Kazakhstan*
[2] *International University of Engineering and Technology, Almaty, Kazakhstan*
[3] *Kh. Dosmukhamedov Atyrau University, Atyrau, Kazakhstan*
[4] *Dalian University of Technology, Dalian, China*
✉ *Correspondent author: jomartova@mail.ru*

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With the rapid development of digital technologies and automation of production processes, food industry enterprises are increasingly focusing on improving production management efficiency. Improving production planning is a key area, as the efficient use of resources, timely order fulfillment, cost reduction, and increased competitiveness depend on the quality of production schedules. This task is particularly challenging for bakeries, where the production cycle includes numerous interrelated operations, the execution of which requires strict adherence to time and resource constraints.

Bakery production is a multi-stage process, including raw material preparation, dough preparation, product molding, proofing, baking, cooling, and packaging of finished products. Each of these operations is performed on specific equipment and has its own technological requirements. Furthermore, various production orders can simultaneously compete for the enterprise's limited resources, significantly complicating the planning process. In such circumstances, traditional scheduling methods often prove ineffective, especially as production volumes and the number of orders increase.

This study examines the application of the CP-SAT (Constraint Programming Satisfiability) method to solving the problem of optimal production process planning at a bakery. This method is a modern combinatorial optimization tool and enables efficient solutions to problems containing a large number of interrelated constraints. CP-SAT allows for the consideration of process characteristics, equipment constraints, operational sequence requirements, and time dependencies between individual production stages.

To solve this problem, a mathematical production planning model was developed that reflects the structure of a bakery's production process. The model takes into account production operations, available resources, job durations, and equipment operating conditions. Additionally, the possibility of parallel execution of individual processes is considered, allowing for more efficient utilization of available production capacity. The objective function of the model is to minimize the overall production plan execution time, thereby reducing product manufacturing time and improving plant efficiency.

The practical implementation of the proposed approach was accomplished using the Google OR-Tools library, which contains a high-performance CP-SAT solver. Based on the developed model, an algorithm for automated production scheduling was created. Experiments demonstrated the method's ability to effectively account for numerous process constraints and generate high-quality production plans within a reasonable computation time. The results confirm the potential of constraint programming methods for solving production planning problems in the bakery industry.

Thus, the use of CP-SAT technology improves the quality of production process planning, ensures more efficient equipment utilization, and reduces order fulfillment time. The proposed approach can serve as the basis for developing intelligent decision support systems aimed at automating production management and increasing the efficiency of food industry enterprises.

Keywords: optimization, planning, production, bread, Python programming, CP-SAT.

CP-SAT ӘДІСІНЕ НЕГІЗДЕЛГЕН НАН-ТОҚАШ ӨНІМДЕРІ КӘСІПОРНЫНЫҢ ӨНДІРІСТІК ПРОЦЕСІН ОҢТАЙЛАНДЫРУ

Т.Ж. Мазаков[1,2], Ш.А.Джомартова[1]✉, Ә.Т.Мазақова[1], Т.Т. Дузбаев[1], А.Д. Майлыбаева[3], Б.Чжао[4]

[1] *Әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан*
[2] *Халықаралық инженерлік-технологиялық университеті, Алматы, Қазақстан*
[3] *Kh. Досмұхамедов атындағы Атырау университеті, Атырау, Қазақстан*
[4] *Далянь технологиялық университеті, Далянь, Қытай*
✉ *e-mail: jomartova@mail.ru*

Сандық технологиялардың және өндірістік процестерді автоматтандырудың қарқынды дамуымен тамақ өнеркәсібі кәсіпорындары өндірісті басқару тиімділігін арттыруға көбірек көңіл бөлуде. Өндірісті жоспарлауды жетілдіру маңызды бағыт болып табылады, себебі ресурстарды тиімді пайдалану, тапсырыстарды уақтылы орындау, шығындарды азайту және бәсекеге қабілеттілікті арттыру өндіріс кестелерінің сапасына байланысты. Бұл міндет, әсіресе, өндіріс циклі уақыт пен ресурстардың шектеулерін қатаң сақтауды талап ететін көптеген өзара байланысты операцияларды қамтитын наубайханалар үшін қиын.

Нан пісіру өндірісі - шикізатты дайындау, қамыр дайындау, өнімді қалыптау, герметизациялау, пісіру, салқындату және дайын өнімді орау сияқты көп сатылы процесс. Бұл операциялардың әрқайсысы нақты жабдықта орындалады және өзіндік технологиялық талаптарға ие. Сонымен қатар, әртүрлі өндірістік тапсырыстар бір уақытта кәсіпорынның шектеулі ресурстары үшін бәсекелесе алады, бұл жоспарлау процесін айтарлықтай қиындатады. Мұндай жағдайларда дәстүрлі жоспарлау әдістері көбінесе тиімсіз болып шығады, әсіресе өндіріс көлемі мен тапсырыстар саны артқан сайын.

Бұл зерттеуде наубайханада өндірістік процесті оңтайлы жоспарлау мәселесін шешу үшін CP-SAT (шектеу бағдарламалау қанағаттанушылығы) әдісін қолдану қарастырылады. Бұл әдіс заманауи комбинаторлық оңтайландыру құралы болып табылады және көптеген өзара байланысты шектеулерді қамтитын мәселелерді тиімді шешуге мүмкіндік береді. CP-SAT технологиялық сипаттамаларды, жабдық шектеулерін, операциялық тізбек талаптарын және жеке өндіріс кезеңдерінің арасындағы уақыт тәуелділіктерін ескеруге мүмкіндік береді.

Бұл мәселені шешу үшін наубайхананың өндірістік процесінің құрылымын көрсететін математикалық өндірісті жоспарлау моделі жасалды. Модель өндірістік операцияларды, қолжетімді ресурстарды, жұмыс ұзақтығын және жабдықтың жұмыс жағдайларын ескереді. Сонымен қатар, жеке процестерді параллель орындау мүмкіндігі қарастырылады, бұл қолжетімді өндірістік қуатты тиімдірек пайдалануға мүмкіндік береді. Модельдің мақсатты функциясы – өндіріс жоспарының жалпы орындалу уақытын азайту, осылайша өнімді өндіру уақытын қысқарту және зауыттың тиімділігін арттыру.

Ұсынылған тәсілді практикалық түрде енгізу жоғары өнімді CP-SAT шешушісі бар Google OR-Tools кітапханасын пайдалану арқылы жүзеге асырылды. Әзірленген модель негізінде автоматтандырылған өндірісті жоспарлау алгоритмі жасалды. Тәжірибелер әдістің көптеген технологиялық шектеулерді тиімді есепке алу және ақылға қонымды есептеу уақытында жоғары сапалы өндіріс жоспарларын жасау мүмкіндігін көрсетті. Нәтижелер нан-тоқаш өнеркәсібіндегі өндірісті жоспарлау мәселелерін шешу үшін шектеулерді бағдарламалау әдістерінің әлеуетін растайды.

Осылайша, CP-SAT технологиясын пайдалану өндіріс процесін жоспарлау сапасын жақсартады, жабдықты тиімдірек пайдалануды қамтамасыз етеді және тапсырысты орындау уақытын қысқартады. Ұсынылған тәсіл өндірісті басқаруды автоматтандыруға және тамақ өнеркәсібі кәсіпорындарының тиімділігін арттыруға бағытталған интеллектуалды шешім қабылдауды қолдау жүйелерін әзірлеу үшін негіз бола алады.

Түйін сөздер: оңтайландыру, жоспарлау, өндіріс, нан, Python бағдарламалау, CP-SAT.

ОПТИМИЗАЦИИ ПРОИЗВОДСТВЕННОГО ПРОЦЕССА ХЛЕБОПЕКАРНОГО ПРЕДПРИЯТИЯ НА ОСНОВЕ МЕТОДА CP-SAT

Т.Ж.Мазаков[1,2], Ш.А.Джомартова[1]✉, Ә.Т.Мазақова[1], Т.Т.Дузбаев[1], А.Д. Майлыбаева[3], Б.Чжао[4]

[1] *Казахский национальный университет им. Аль-Фараби, Алматы, Казахстан*
[2] *Международный инженерно-технологический университет, Алматы, Казахстан*
[3] *Атырауский университет им. Х. Досмухамедова, Атырау, Казахстан*
[4] *Даляньский технологический университет, Далянь, Китай*
✉ *e-mail: jomartova@mail.ru*

В условиях стремительного развития цифровых технологий и автоматизации производственных процессов предприятия пищевой промышленности все больше внимания уделяют вопросам повышения эффективности управления производством. Одним из важнейших направлений является совершенствование производственного планирования, поскольку от качества составления производственных графиков зависят рациональность использования ресурсов, своевременное выполнение заказов, снижение издержек и повышение конкурентоспособности предприятия. Особенно сложной данная задача является для хлебопекарных производств, где технологический цикл включает множество взаимосвязанных операций, выполнение которых требует строгого соблюдения временных и ресурсных ограничений.

Производство хлебобулочной продукции представляет собой многоэтапный процесс, включающий подготовку сырья, приготовление теста, формование изделий, расстойку, выпекание, охлаждение и упаковку готовой продукции. Каждая из этих операций выполняется на определенном оборудовании и имеет собственные технологические требования. При этом различные производственные заказы могут одновременно конкурировать за ограниченные ресурсы предприятия, что существенно усложняет процесс планирования. В таких условиях традиционные методы составления расписаний зачастую оказываются недостаточно эффективными, особенно при увеличении объемов производства и количества заказов.

В настоящем исследовании рассматривается применение метода программирования в ограничениях CP-SAT (Constraint Programming Satisfiability) для решения задачи оптимального планирования производственных процессов на хлебопекарном предприятии. Данный метод относится к современным инструментам комбинаторной оптимизации и позволяет находить эффективные решения для задач, содержащих большое количество взаимосвязанных ограничений. Использование CP-SAT предоставляет возможность учитывать особенности технологического процесса, ограничения по оборудованию, требования к последовательности выполнения операций и временные зависимости между отдельными этапами производства.

Для решения поставленной задачи разработана математическая модель производственного планирования, отражающая структуру технологического процесса хлебопекарного предприятия. В модели учитываются производственные операции, доступные ресурсы, продолжительность выполнения работ и условия использования оборудования. Дополнительно рассматривается возможность параллельного выполнения отдельных процессов, что позволяет более полно использовать имеющиеся производственные мощности. Целевой функцией модели является минимизация общей продолжительности выполнения производственного плана, что способствует сокращению времени изготовления продукции и повышению эффективности работы предприятия.

Практическая реализация предложенного подхода выполнена с использованием библиотеки Google OR-Tools, содержащей высокопроизводительный решатель CP-SAT. На основе разработанной модели создан алгоритм автоматизированного формирования производственного расписания. Проведенные эксперименты продемонстрировали способность метода эффективно учитывать многочисленные технологические ограничения и формировать качественные планы производства за приемлемое время вычислений. Полученные результаты подтверждают перспективность использования методов программирования в ограничениях для решения задач производственного планирования в хлебопекарной отрасли.

Таким образом, применение технологии CP-SAT позволяет повысить качество планирования производственных процессов, обеспечить более эффективное использование оборудования и сократить временные затраты на выполнение заказов. Предложенный подход может служить основой для разработки интеллек-

туальных систем поддержки принятия решений, ориентированных на автоматизацию управления производством и повышение эффективности предприятий пищевой промышленности.

Ключевые слова: оптимизация, планирование, производство, хлеб, программирование на Python, CP-SAT.

Introduction. Modern food industry enterprises operate in a highly competitive environment, facing increasing demands on product quality and the need to improve resource efficiency. In the bakery industry, these requirements are particularly important due to the limited shelf life of products, the continuous production process, and the need to adhere to technological standards.

The production process at a bakery involves several sequential stages: raw material preparation, dough mixing, dividing and shaping, proofing, baking, and cooling the finished product [1]. Each stage requires specialized equipment with limited throughput. Failure to coordinate these operations leads to equipment downtime, increased energy consumption, and reduced product quality [2-4].

The problem of production planning is to determine the optimal order of production operations while observing technological and resource constraints [5-7]. This problem belongs to the class of NP-hard combinatorial optimization problems and requires the use of specialized solution methods [8, 9].

One of the most promising approaches is constraint programming (CP), which combines the capabilities of logical inference and mathematical optimization [10, 11]. The modern CP-SAT solver, implemented as part of the Google OR-Tools library, demonstrates high efficiency in solving complex resource planning and allocation problems [12-14].

The aim of this work is to develop a mathematical model of production planning for a bakery enterprise and to study the possibilities of using the CP-SAT method to minimize the overall time of execution of the production plan.

To solve such problems, mixed integer linear programming methods are used, which allow obtaining optimal solutions for small-scale problems [15].

An alternative approach is constraint programming (CP), based on the description of the admissible states of the system through a set of logical and arithmetic constraints, which has shown high efficiency in solving problems of calendar planning, resource allocation and scheduling [16].

In recent years, considerable attention has been paid to the CP-SAT solver, which combines constraint programming methods with modern SAT technologies. Research shows the successful use of CP-SAT for problems in production planning, logistics, and supply chain management.

Despite the existence of a large number of studies in the field of production planning, the application of the CP-SAT method to bakery production problems has not been sufficiently studied [17]. This determines the relevance of this study.

Materials and methods. The article examines a bakery that produces several types of products. Each production order goes through a sequence of technological stages shown in Figure 1: 1) raw material preparation; 2) dough kneading; 3) molding; 4) proofing; 5) baking; 6) cooling; and 7) packaging.

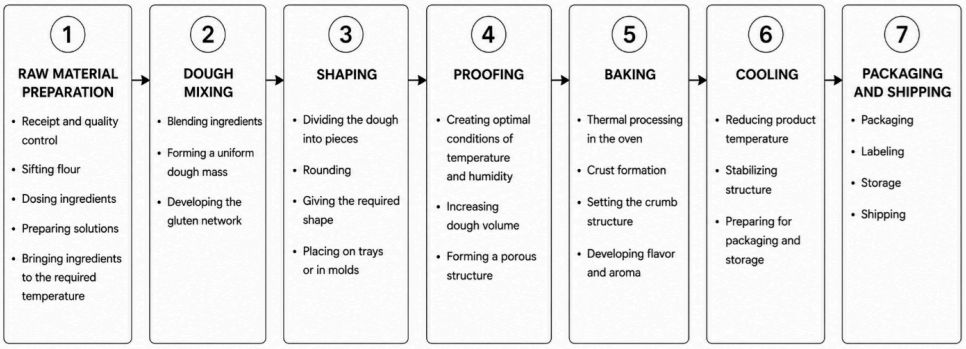


Fig. 1 - Scheme of the technological process of bakery production

1 stage: Raw Material Preparation. The technological cycle of bakery production begins with the preparation of raw materials, which includes their acceptance, quality control, and pre-processing. The

main ingredients used are flour, water, yeast, salt, sugar, and vegetable fats, while various food additives are used to regulate the technological properties of the product.

The raw material preparation stage includes a set of technological procedures aimed at ensuring the required characteristics of the starting components before the start of the production process. This stage includes incoming raw material quality control, flour cleaning and aeration through sifting, ingredient dosing according to the technological recipe, preparation of auxiliary solutions, and bringing the raw materials to the established temperature conditions necessary for the stable operation of subsequent technological operations.

The quality of raw material preparation significantly impacts the dough properties, process stability, and finished product quality. Errors in component dosing can alter the dough structure, degrade flavor, and reduce yield.

From a production planning perspective, the raw material preparation stage is the initial operation of the process flow, which must be completed before dough mixing begins. Insufficient raw material preparation can delay subsequent operations and lead to reduced equipment efficiency.

2 stage: Mixing. At this stage, flour, water, yeast, salt and other ingredients are mixed in a dough mixing machine until a homogeneous mass with the required rheological properties is obtained. The main tasks of this stage are: 1) uniform distribution of ingredients; 2) formation of a gluten framework; 3) saturation of the dough with oxygen; 4) preparation of the dough for further processing. The kneading time usually ranges from 15 to 40 minutes, depending on the recipe and type of product.

3 stage: Formation. After kneading, the dough is divided into pieces of the required weight and shape. During the shaping stage, the product is given the required geometric parameters. Basic operations: 1) dividing the dough into portions; 2) rounding the pieces; 3) giving the final shape; 4) placing in molds or on sheets. The quality of the moulding has a direct impact on the appearance and uniformity of the finished product.

4 stage: Proofing. After forming, the dough pieces are placed in proofing cabinets, where special temperature and humidity conditions are maintained. During proofing, the following processes occur: 1) the yeast continues to ferment; 2) carbon dioxide is released; 3) the dough increases in volume; 4) the porous structure of the future bread is formed; 5) the taste and aroma of the product improve. Proofing is typically carried out at a temperature of 30-40°C and a relative humidity of 75-85%. The duration depends on the recipe and type of product and can range from 20 to 90 minutes.

5 stage: Baking. Baking is the thermal processing of dough pieces in bakery ovens. During baking, the following processes occur: the final increase in the volume of the product; crust formation; strengthening of the crumb structure; removal of excess moisture; formation of the taste and aroma of the product. Furnaces are one of the most expensive and limited resources of an enterprise, so their loading significantly affects production efficiency.

6 stage: Cooling. After baking, the products are transferred to the cooling section. At this stage, the product temperature is reduced to levels suitable for packaging and storage. The main tasks of cooling is stabilization of the product structure and prevention of condensation formation. Insufficient cooling can result in deterioration of product quality and reduced shelf life.

7 stage: Packaging and Shipping. Packaging and shipping of products is the final stage of the bakery production process. After cooling, finished products are sent to the packaging area, where they are prepared for storage, transportation, and distribution. At this stage, the following operations are performed: 1) quality control of finished products; 2) sorting products by type and batch; 3) packaging products in individual or group containers; 4) applying markings and information about the product; 5) forming transport batches; 6) placing products in the finished goods warehouse; 7) preparing documents for shipment; 8) transferring products to consumers or trading organizations.

The main goals of this stage are to preserve the quality and freshness of bakery products; protect them from mechanical damage and external influences; ensure traceability of product batches; and organize timely delivery to consumers. For bakeries, this stage is especially important, as most of their products are perishable. Delays during packaging or shipping can lead to a shorter shelf life and increased production losses. Each stage may use one or more parallel devices.

Results and discussion. *Theoretical results.* It is necessary to determine: 1) the sequence of execution of product batches; 2) the purpose of operations on equipment; 3) the start and end times of operations.

The goal is to minimize the overall lead time of the production plan. Let us introduce the following notations:

$I = \{1, 2, ..., n\}$ – many types of products,

$J = \{1, 2, ..., m\}$ – many stages of the production process,

$K_i = \{1, 2, ..., K_i\}$ – a set of product lots $i \in I$.

R_j – a set of resources (machines) available at stage $j \in J$.

Technological parameters: $p_{ij} \in \mathbb{Z}^+$ – processing time of a unit of product $i$at the stage j(min), $q \in \mathbb{Z}^+$ – the size of a standard batch (units of production), $b_i \in \mathbb{Z}^+$ — order volume for the product i(units), $K_i = \lceil \frac{b_i}{q} \rceil$ – number of product batches i.

Resource constraints: $M_j \in \mathbb{Z}^+$ – the number of cars at the stage j, $\mu_{jr} \in \{0, 1\}$ – indicator of resource availability $r$at the stage j.

Time limits: $W_{jj'} \geq 0$ – minimum waiting time between stages j and j', $\Delta_{jj'} \geq 0$ – the maximum allowable delay between stages j and j', $T^{\max} \in \mathbb{Z}^+$ – planning horizon.

Key variables: $S_{ijk} \in \mathbb{Z}^+$ – the start time of processing a batch k of product i at stage j, $E_{ijk} \in \mathbb{Z}^+$ – the end time of processing of a batch k of product i at stage j, $X_{ijkr} \in \{0, 1\}$ – a binary variable equal to 1 if the batch (i, k) at the stage j is assigned to the resource r, and 0 otherwise.

Auxiliary variables: $C_{\max} \in \mathbb{Z}^+ -$ total production completion time, $Z_{ijk,i'j'k'} \in \{0, 1\} -$ binary variable of the order of operations.

The main goal is to minimize the total production time:

$$T_{\min} = \min C_{\max} \tag{1}$$

where $C_{\max} = \max_{i \in I, k \in K_i} E_{i,m,k}$ (the end time of the last operation of the last batch at the last stage m).

Restrictions. Relationship between the start and end times of operations:

$$E_{ijk} = S_{ijk} + p_{ij} \cdot q \forall i \in I, j \in J, k \in K_i. \tag{2}$$

(For incomplete lots, $q$it is replaced by the actual size).

Sequence of operations within one batch:

$$S_{i,j+1,k} \geq E_{ijk} + W_{j,j+1} \quad \forall i \in I, j \in J \setminus \{m\}, k \in K_i \tag{3}$$

Maximum delay limit:

$$S_{i,j+1,k} \leq E_{ijk} + \Delta_{j,j+1} \quad \forall i \in I, j \in J \setminus \{m\}, k \in K_i. \tag{4}$$

Resource Assignment: Each operation must be assigned to exactly one machine.

$$\sum_{r \in R_j} X_{ijkr} = 1 \forall i \in I, j \in J, k \in K_i. \tag{5}$$

Cumulative resource constraint (non-overlapping operations): No more than one operation can be executed on each machine at any given time.

$$\sum_{i \in I} \sum_{k \in K_i} X_{ijkr} \cdot I\left[S_{ijk} \leq t < E_{ijk}\right] \leq 1 \forall j \in J, r \in R_j, t \in [0, T^{\max}], \tag{6}$$

where $I[\cdot]$ is the indicator function. In MILP/CP models, this constraint is formulated through disjunctions: for any two operations (i, k) and (i', k') assigned to the same machine r at stage, either j or $E_{i'jk'} \leq S_{ijk}$ must hold $E_{ijk} \leq S_{i'jk'}$.

Determining the total production time:

$$C_{\max} \geq E_{i,m,k} \forall i \in I, k \in K_i. \tag{7}$$

Time limits:

$$0 \leq S_{ijk} \leq T^{\max}, 0 \leq E_{ijk} \leq T^{\max} \forall i, j, k. \tag{8}$$

To carry out numerical calculations in Python, a program for calculating process parameters has been implemented.

Numerical experiments. To evaluate the effectiveness of the developed model, computational experiments were conducted on a set of test production scenarios simulating the operation of a bakery with varying workloads. The experimental calculations were performed using the Python programming language and the Google OR-Tools library, which contains an implementation of the CP-SAT solver.

The study examined production plans of varying sizes, encompassing between 50 and 120 batches. Each batch underwent a complete production cycle, consisting of raw material preparation, dough mixing, molding, proofing, baking, cooling, and packaging.

Standard operating time values were set for each stage, as well as resource availability constraints.

The overall time to complete the production plan (makespan), defined as the completion time of the last technological operation among all production batches, was used as the efficiency criterion.

The following assumptions were made to conduct the experiments:

- production resources are available throughout the entire planning horizon;
- no equipment readjustment is required between batches;
- the duration of technological operations is deterministic;
- each operation is performed without interruption;
- the technological sequence of operations is mandatory for all types of products.

The results of the computational experiments are presented in Table 1.

Table 1 - Optimization results

Amount	Makespan	Solution time
50	1340	1.2
70	1865	2.8
90	2417	4.6
120	3175	8.1

An analysis of the obtained results shows a consistent increase in the overall production plan execution time as the number of production batches increases. For example, as production volume increases from 50 to 120 batches, the makespan value increases from 1340 to 3175 minutes. At the same time, the solver execution time increases from 1.2 to 8.1 seconds.

It should be noted that the increase in computational time is significantly slower than the increase in problem size. This result demonstrates the high computational efficiency of the CP-SAT method and its suitability for solving practical production planning problems in a real-world enterprise.

To visually present the optimization results, a Gantt chart was constructed, reflecting the sequence of technological operations for one production batch (Figure 2).

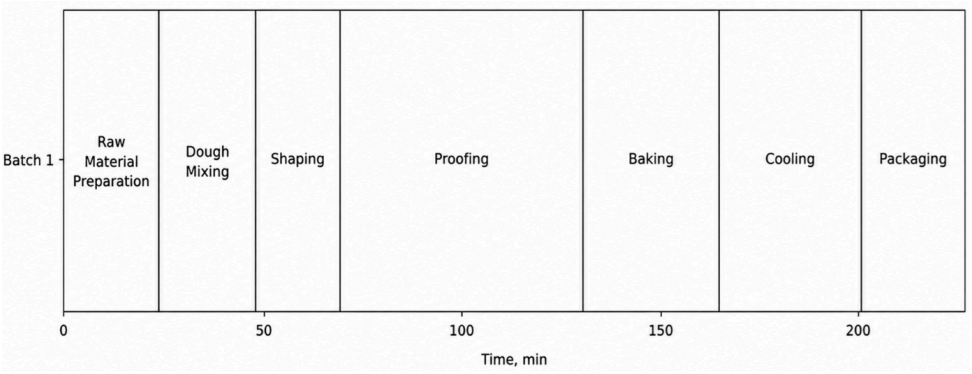


Fig. 2 - Gantt chart

The diagram shows that the production process strictly follows the flow chart (Figure 3). Each operation begins only after the previous stage is completed, ensuring compliance with bakery production requirements. The proofing stage takes the longest time, reflecting the actual conditions of bakery production.

To assess the use of production resources, an equipment loading diagram was constructed.

An analysis of equipment utilization shows that the proofing, baking, and cooling sections are the most intensively utilized. These stages are critical in terms of production system throughput and have a significant impact on the overall production cycle time.

Additionally, a comparative assessment of the duration of technological operations for various types of products was carried out (Figure 4).

The data presented demonstrates that different types of products are characterized by different durations of individual processing stages. The greatest differences are observed during the proofing and baking stages, due to the specific recipes and production processes used to produce these baked goods.

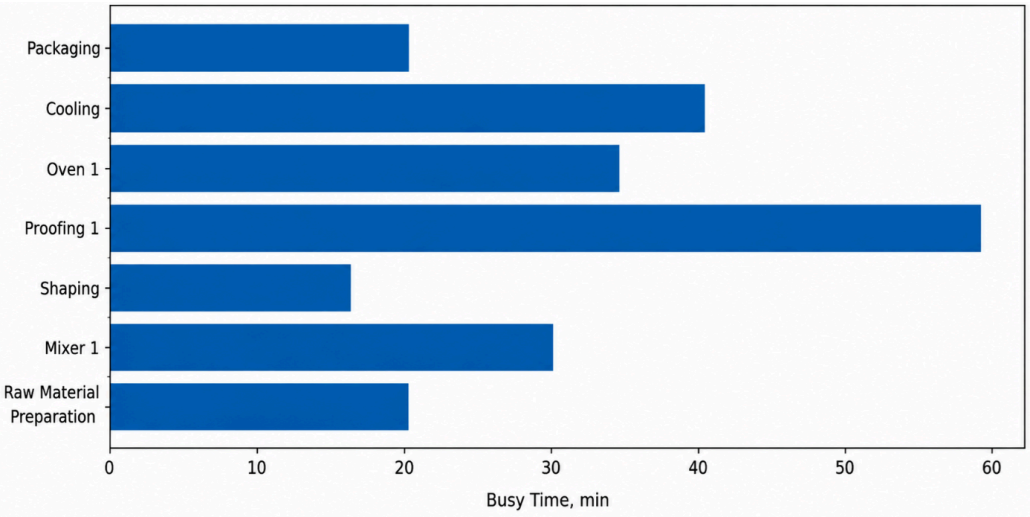


Fig. 3 - Bakery production workload diagram

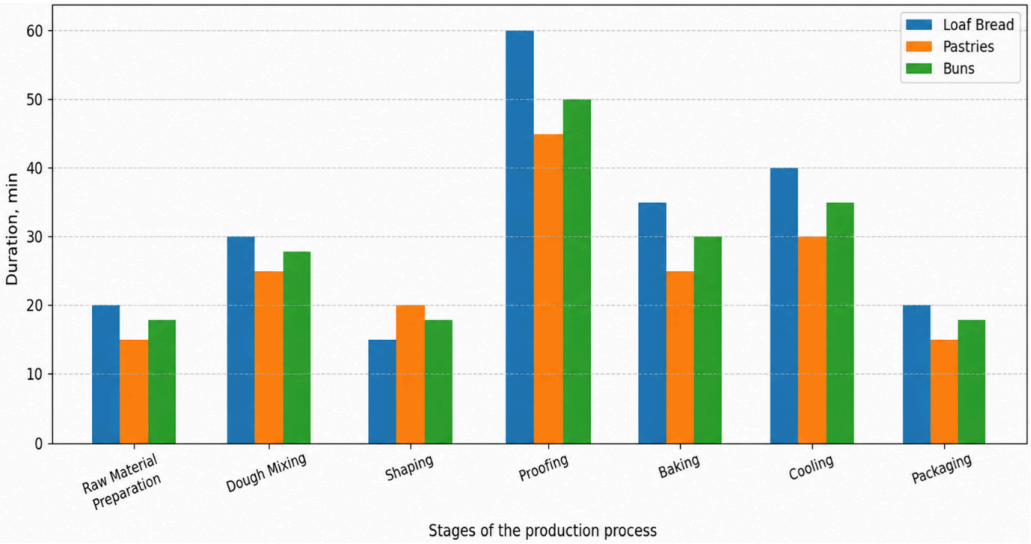


Fig. 4 - Duration of technological operations by type of product

Thus, the results of the numerical experiments confirm the viability of the developed mathematical model and the effectiveness of the CP-SAT method for solving production planning problems in bakeries. The resulting schedules ensure the efficient use of production resources, reduce equipment downtime, and minimize overall order fulfillment time.

The obtained results confirm the effectiveness of the CP-SAT constraint programming method for solving production planning problems in bakeries. Computational experiments demonstrated that the proposed model enables the generation of feasible and high-quality production schedules while adhering to the technological constraints typical of bakery production.

An analysis of the results showed that with an increase in the number of production batches, there is a consistent increase in the objective function value and computation time. However, the increase in computational costs is significantly slower than the increase in problem size, demonstrating the good scalability of the approach used. Even for the largest experimental scenarios, the solution search time remains acceptable for practical use in operational production planning systems.

One of the key advantages of the CP-SAT method is its ability to directly account for complex logical and resource constraints. Unlike classical mixed-integer linear programming (MILP) models, which require the introduction of numerous additional binary variables

and constraints, CP-SAT allows for a more natural description of production processes using interval variables, sequence constraints, and equipment usage rules. This is particularly relevant for bakery production, where strict time dependencies and equipment availability constraints exist between individual process operations.

The generated schedules ensure a more even distribution of workload among production resources. Analysis of equipment utilization charts shows that key production capacities are utilized without significant downtime, and the most critical sections of the production line do not become bottlenecks in the production process. This increases equipment utilization and reduces lost production time.

It should be noted that the proposed model is based on a deterministic description of the production environment. In real-world conditions, the efficiency of the production process can be affected by random factors, such as deviations in the duration of process operations, unscheduled equipment shutdowns, delays in raw material deliveries, and changes in order volumes. Therefore, the obtained results should be considered a baseline scenario for enterprise operation under known production parameters.

Conclusions. This article examines the problem of optimizing production planning for a bakery using the CP-SAT constraint programming method. A mathematical model has been developed that takes into account the sequence of process operations, production resource constraints, time dependencies between product processing stages, and the operational characteristics of bakery equipment.

Based on the developed model, a solution algorithm was implemented using the Google OR-Tools library. Computational experiments confirmed the feasibility of effectively generating production schedules for problems of varying dimensions. The results demonstrated that the CP-SAT method minimizes the overall production plan execution time, promotes a more even distribution of equipment load, and reduces time losses associated with process line downtime.

Unlike traditional approaches based on mixed integer programming, the proposed method demonstrates high computational efficiency when solving problems with a large number of operations and constraints. This makes it a promising tool for supporting decision-making in modern digital manufacturing.

The developed model can serve as the basis for creating intelligent production management systems at food industry enterprises. Its application ensures increased resource efficiency, improved production planning, and reduced operating costs through optimized production schedules.

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Сведения об авторах

Мазаков Т.Ж.- д.ф.-м.н., профессор, Казахский национальный университет им. аль-Фараби, Международный инженерно-технологический университет, Алматы, Казахстан, e-mail: tmazakov@mail.ru;

Джомартова Ш.А.- д.т.н., профессор, Казахский национальный университет им. аль-Фараби, Алматы, Казахстан, e-mail: jomartova@mail.ru;

Мазақова Ә.Т. - PhD, старший преподаватель, Казахский национальный университет им. аль-Фараби, Алматы, Казахстан, Алматы, e-mail: aigerym97@mail.ru;

Дузбаев Т.Т.- преподаватель, Казахский национальный университет им. аль-Фараби, Алматы, Казахстан, e-mail: talgat.duzbaev@mail.ru;

Майлыбаева А.Д. - к.ф.-м.н., доцент, Атырауский университет и. Х. Досмухамедова, Атырау, Казахстан, e-mail: a.maylibayeva@asu.edu.kz;

Чжао Байдун - старший преподаватель, Даляньский политехнический университет, Китай, e-mail: xiaoyuezhishang@vip.qq.com.

Information about the authors

Mazakov T. Zh. - Doctor of Physical and Mathematical Sciences, Professor, Al-Farabi Kazakh National University,

International University of Engineering and Technology Almaty, Kazakhstan, e-mail: tmazakov@mail.ru;

Jomartova Sh. A.- Doctor of Technical Sciences, Professor, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: jomartova@mail.ru;

Mazakova A.T. - PhD, senior lecturer, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: aigerym97@mail.ru;

Duzbaev T.T.- teacher, Al-Farabi Kazakh National University, Almaty, Kazakhstan, e-mail: talgat.duzbaev@mail.ru;

Mailybayeva A.D. - PhD, Associate Professor, Kh. Dosmukhambetov Atyrau University named after, Atyrau, Kazakhstan, e-mail: a.maylibayeva@asu.edu.kz;

Zhao Baidong - Senior Lecturer, Dalian Polytechnic University, China, e-mail: xiaoyuezhishang@vip.qq.com.